Exponential Stabilization for Stochastic Networked Systems with Interval Distribution Time Delays

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***Abstract*—To study the problem of exponential stabilization for stochastic networked systems with interval distribution time delays. A new approach is given to model the networked control systems with the stochastic time delays which is assumed to be satisfying a interval distribution. The mean-square exponential stabilization condition is presented in terms of linear matrix inequality. A numerical example is given to demonstrate the validity of the results.**

***Index Terms*—Interval Distribution, Networked systems, Stochastic delays, Linear matrix inequality (LMI)**

I. Introduction

Networked control systems (NCSs) are a type of distributed control systems where sensors, actuators and controllers are interconnected by communication networks. The introduction of common-bus network architectures can improve the efficiency flexibility, and reliability of these integrated applications, reducing installation, reconfiguration and maintenance time and cost. For these advantages, the networked control systems receive more and more attention and has been a very hot research topic [1-2].

The insertion of the communication network in the feedback control loop makes the analysis and design of the NCSs very complex [3-4]. The change of communication architecture from point-to-point to common bus induces different forms of time delay uncertainty between sensors, actuators and controllers. These time delays come from the time sharing of the communication medium as well as the computation time required for physical signal coding and communication processing [5-6]. It is well known in control systems that time delays can degrade a system's performance and even cause system instability. Another significant difference between NCSs and standard digital control is the possibility that data may be lost while in transit through the network because of uncertainty and noise. To analyze the above mentioned issues, especially the problem of network induced delay and packet dropout, Gao presents a new approach to solve the problem of stabilization for networked control systems. A controller design procedure is proposed for stabilization of the closed-loop NCSs [7]. In [8], the stabilization problem for a class of uncertain networked control systems with random communication network induced delays is considered. Based on the Lyapunov method, a dynamic output feedback controller is designed in terms of the solvability of linear matrix inequalities. Wang studies the problem of designing H_∞ controllers for networked control systems with both network induced time delay and packet disordering [9]. A delay switching based method is proposed to model the NCSs with long time delay as switched systems. And H_∞ controller design is proposed by using LMI. A new Lyapunov-Krasovskii functional, which makes use of the information of both the lower and upper bounds of the time varying network induced delay, is proposed to drive a new delay-dependent H_∞ stabilization criterion [10].

But the above papers consider the robust control of certain NCSs. In this paper, our objective is to consider the problem of mean-square exponential stability control for a class of networked control systems with interval distribution time delay. The mean-square exponential stability condition is obtained by using the LMI approach.

II. Problem Formulation

Consider the following control system with delay

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) \\ &x(t) = \phi(t) \qquad\qquad t \in [-d,\ 0] \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, d is state delay $A, A_d \in R^{n\times n}$ are known real constant matrices, $B \in R^{n\times m}$ is input matrix, $\phi(t) \in R^n$ is the given initial state on $[-d,\ 0]$.

Throughout this note, we suppose that all the system's states are available for a state feedback control. In the presence of the control network, data transfers between the controller and the remote system, e.g., sensors and actuators in a distributed control system will induce network delay in addition to the controller proceeding delay. We introduce stochastic delay $\tau(t)$ to denote the network-induced delay.

We will design the state feedback controller

$$u(t) = Kx(t - \tau(t)) \tag{2}$$

Inserting the controller(2) into system (1), we obtain the closed system:

$$\begin{aligned}&\dot{x}(t) = Ax(t) + A_d x(t-d) + BKx(t-\tau(t)) \\ &x(t) = \psi(t) \qquad t \in [-\bar{d},\ 0]\end{aligned} \tag{3}$$

The initial condition of the state is supplemented as $x(t) = \psi(t)$, where $\psi(t)$ is a smooth function on $[-\bar{d},\ 0]$, $\bar{d} = \max\{\tau,\ d\}$ .Therefore, there exists a positive constant $\bar{\psi}$ satisfying

$$\|\dot{\psi}(t)\| \le \bar{\psi} \quad t \in [-\bar{d},\ 0]$$

It is assumed that there exists a constant $\tau_1 \in [0,\ \tau]$ such that the probability of $\tau(t)$ taking values in $[0,\ \tau_1)$ and $[\tau_1,\ \tau]$ can be observed. In order to employ the information of the probability distribution of the delays, the following sets are proposed firstly

$$\Omega_1 = \{t : \tau(t) \in [0,\ \tau_1)\},\quad \Omega_2 = \{t : \tau(t) \in [\tau_1,\ \tau]\}$$

Obviously, $\Omega_1 \bigcup \Omega_2 = R^+$ and $\Omega_1 \bigcap \Omega_2 = \Phi$

Then we define two functions as:

$$h_1(t) = \begin{cases}\tau(t) & t \in \Omega_1 \\ 0 & t \notin \Omega_1\end{cases},\quad h_2(t) = \begin{cases}\tau(t) & t \in \Omega_2 \\ \tau_1 & t \notin \Omega_2\end{cases} \tag{4}$$

Corresponding to $\tau(t)$ taking values in different intervals, a stochastic variable $\beta(t)$ is defined

$$\beta(t) = \begin{cases}1 & t \in \Omega_1 \\ 0 & t \in \Omega_2\end{cases} \tag{5}$$

Where we suppose that $\beta(t)$ is a Bernoulli distributed sequence with $\Pr ob\{\beta(t) = 1\} = E\{\beta(t)\} = \beta$, where $\beta \in [0,1]$ is a constant.

With (4-5),we know that the systems (3) is equivalent to

$$\begin{aligned}\dot{x}(t) &= Ax(t) + A_d x(t-d) + \beta(t)BKx(t-h_1(t)) \\ &\quad + (1-\beta(t))BKx(t-h_2(t)) \\ &= \bar{A}\xi(t) \\ x(t) &= \psi(t) \qquad t \in [-\bar{d},\ 0]\end{aligned} \tag{6}$$

where

$$\bar{A} = [A\ \ A_d\ \ \beta(t)BK\ \ (1-\beta(t))BK]$$

$$\xi^T(t) = [x^T(t),\ \ x^T(t-d),\ \ x^T(t-h_1(t)),\ \ x^T(t-h_2(t))]$$

III. Main Results

Lemma1[2] For any vectors a, b and matrices N, X, Y, Z with appropriate dimensions, if the following matrix inequality holds

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$$

then we have

$$-2a^T Nb \le \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Lemma2[11] The LMI $\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$ is equivalent to

$$R(x) > 0,\ Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Theorem1 For the given constants $\alpha > 0, 1 \ge \beta \ge 0$, if there exist positive-definite matrices $P, Q, R \in R^{n\times n}$ and matrices $K \in R^{m\times n}$, X, Y with appropriate dimensions, such that the following matrix inequality holds

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{bmatrix} < 0 \tag{7}$$

where

$$\Theta_{11} = \begin{bmatrix} \begin{array}{l} PA + A^T P + Q + 2\alpha P \\ +\tau_2 X_{11} + \tau_2 A^T RA \end{array} & P\bar{A}_d + \tau_2 X_{12} + \tau_2 A^T RA_d \\ * & -e^{-2\alpha d}Q + \tau_2 X_{22} + \tau_2 A_d^T RA_d \end{bmatrix}$$

$$\Theta_{12} = \begin{bmatrix} P\beta BK + Y_1 + \tau_2 X_{13} + \tau_2 A^T R\beta BK & P(1-\beta)BK + \tau_2 X_{14} - Y_1 + \tau_2 A^T R(1-\beta)BK \\ Y_2 + \tau_2 X_{23} + \tau_2 A_d^T R\beta BK & \tau X_{24} - Y_2 + \tau_2 A_d^T R(1-\beta)BK \end{bmatrix}$$

$$\Theta_{22} = \begin{bmatrix} \tau_2 X_{33} + Y_3 + Y_3^T + \tau_2 K^T B^T R\beta BK & -Y_3 + Y_4^T + \tau_2 X_{34} \\ * & \tau_2 X_{44} - Y_4 - Y_4^T + \tau_2 K^T B^T R(1-\beta)BK \end{bmatrix}$$

with the controller (2), the network control systems(6) is mean-square exponentially stable.

Proof Choose a Lyapunov functional candidate for the system (6) as follows：

$$V(t) = x^T(t)Px(t) + \int_{t-d}^{t} x^T(s)Qe^{2\alpha(s-t)}x(s)ds + \int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)dsd\theta$$

where P, Q, R positive-definite matrices in theorem1. Then, along the solution of system (6) we have

$$\begin{aligned}&\dot{V}(t) + 2\alpha V(t) \\ &= 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) \\ &\quad - x^T(t-d)Qe^{-2\alpha d}x(t-d) \\ &\quad + \tau\dot{x}^T(t)R\dot{x}(t) + 2\alpha x^T(t)Px(t) \\ &\quad - \int_{t-\tau_2}^{t} \dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds\end{aligned} \tag{8}$$

With

$$x(t-h_1(t)) - x(t-h_2(t)) - \int_{t-h_2(t)}^{t-h_1(t)} \dot{x}(s)ds = 0$$

For any $4n \times n$ matrix

$$N=\begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}$$

We know

$$0=\xi^T(t)N[x(t-h_1(t))-x(t-h_2(t))-\int_{t-h_2(t)}^{t-h_1(t)}\dot{x}(s)ds] \tag{9}$$

With lemma1 and (9), we obtain

$$\begin{aligned} 0 &\le 2\xi^T(t)N[x(t-h_1(t))-x(t-h_2(t))] \\ &+\int_{t-h_2(t)}^{t-h_1(t)}\begin{bmatrix}\xi(t)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix} X & Y-N \\ Y^T-N^T & Re^{2\alpha(s-t)}\end{bmatrix}\begin{bmatrix}\xi(t)\\ \dot{x}(s)\end{bmatrix}ds \\ &\le 2\xi^T(t)Y[x(t-h_1(t))-x(t-h_2(t))] \\ &+\tau_2\xi^T(t)X\xi(t)+\int_{t-\tau_2}^{t}\dot{x}^T(s)Re^{2\alpha(s-t)}\dot{x}(s)ds \end{aligned} \tag{10}$$

Inserting(10)into(8), we have：

$$\begin{aligned} &\dot{V}(t)+2\alpha V(t) \\ &\le x^T(t)[PA+AP+Q+2\alpha P]x(t)+2x^T(t)PA_d x(t-d) \\ &+2x^T(t)P\beta(t)BKx(t-h_1(t)) \\ &+2x^T(t)P(1-\beta(t))BKx(t-h_2(t)) \\ &-x^T(t-d)Qe^{-2\alpha d}x(t-d) \\ &+2\xi^T(t)Y\begin{bmatrix}0 & 0 & I & -I\end{bmatrix}\xi(t) \\ &+\tau_2\xi^T(t)X\xi(t)+\tau_2\dot{x}^T(t)R\dot{x}(t) \end{aligned} \tag{11}$$

then

$$\begin{aligned} &E\{\dot{V}(t)+2\alpha V(t)\} \\ &\le \sum_i^r\sum_j^r \mu_i(z(t))\mu_j(z(t))\xi^T(t)\Theta\xi(t) \end{aligned}$$

With matrix inequality(7), we know

$$E\{\dot{V}(t)\}<-2\alpha E\{V(t)\}$$

therefore

$$\begin{aligned} E\{V(t)\}<E\{V(0)\}e^{-2\alpha t} &\le [\lambda_{\max}(P)+d\lambda_{\max}(Q) \\ &+\tau\lambda_{\max}(R)\bar{\psi}^2]E\{\|\psi(t)\|^2\}e^{-2\alpha t} \end{aligned} \tag{12}$$

Obviously

$$E\{V(t)\}\ge\lambda_{\min}(P)E\{\|x(t)\|^2\} \tag{13}$$

From(12-13), we obtain

$$\begin{aligned} &E\{\|x(t)\|\} \\ &<\sqrt{\frac{\lambda_{\max}(P)+d\lambda_{\max}(Q)+\tau\lambda_{\max}(R)\bar{\psi}^2}{\lambda_{\min}(P)}}E\{\|\psi(t)\|\}e^{-\alpha t} \end{aligned}$$

With the Lyapunov stability theorem and the above inequality, we know that the system (6) is mean-square exponentially stable.

Theorem2 For the given constants $\alpha>0, 1\ge\beta\ge0$, if there exist positive-definite matrices $\bar{P},\bar{Q},\bar{R}\in R^{n\times n}$ and matrix $\bar{K}\in R^{m\times n}$, $\bar{X},\bar{Y}$ with appropriate dimensions, such that the following linear matrix inequality holds

$$\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} \\ * & \Xi_{22}\end{bmatrix}<0 \tag{14}$$

Where

$$\Xi_{11}=\begin{bmatrix} \begin{matrix}A\bar{P}+\bar{P}A^T+\bar{Q} \\ +2\alpha\bar{P}+\tau_2\bar{X}_{11}\end{matrix} & A_d\bar{P}+\tau_2\bar{X}_{12} & \beta B\bar{K}+\tau_2\bar{X}_{13}+\bar{Y}_1 \\ * & -e^{-2\alpha d}\bar{Q}+\tau_2\bar{X}_{22} & \tau_2\bar{X}_{23}+\bar{Y}_2 \\ * & * & \tau_2\bar{X}_{33}+\bar{Y}_3+\bar{Y}_3^T \end{bmatrix}$$

$$\Xi_{12}=\begin{bmatrix} (1-\beta)B\bar{K}-\bar{Y}_1+\tau_2\bar{X}_{14} & \tau_2\beta\bar{P}A^T & \tau_2(1-\beta)\bar{P}A^T \\ \tau_2\bar{X}_{24}-\bar{Y} & \tau_2\beta\bar{P}A_d^T & \tau_2(1-\beta)\bar{P}A_d^T \\ \tau_2\bar{X}_{34}+\bar{Y}_4^T-\bar{Y}_3 & \tau_2\beta\bar{K}^TB^T & 0 \end{bmatrix}$$

$$\Xi_{12}=\begin{bmatrix} \tau_2\bar{X}_{44}-\bar{Y}_4-\bar{Y}_4^T & 0 & \tau_2(1-\beta)\bar{K}^TB^T \\ & -\tau_2\beta\bar{R} & 0 \\ & & -\tau_2(1-\beta)\bar{R} \end{bmatrix}$$

with the controller $u(t)=\bar{K}\bar{P}^{-1}x(t)$, the systems(6) is mean-square exponentially stable.

Proof: The proof is omitted.

IV. SIMULATION

Consider the networked control systems in the form of (7), where

$$A=\begin{bmatrix}-3 & 1 \\ 0 & -4\end{bmatrix}, A_d=\begin{bmatrix}-1 & 0 \\ 1 & -2\end{bmatrix}, \quad B=\begin{bmatrix}0.2 \\ 0.1\end{bmatrix},$$

$\tau=1, \tau_1=0.5, \beta=0.5, \quad \alpha=0.1, d=0.1$. Solving the linear matrix inequality (14), we can obtain the gain matrix K of the stabilizing controller $u(t)$

$$K=[-0.7645 \;\; 2.5692]$$

From the theorem 2, we know that the systems (6) is mean-square exponentially stable.

V. CONSLUSION

This paper considers the problem of mean-square exponential stability control for a class of networked control systems with interval distribution time delay. Based on the Lyapunov stability theorem, a sufficient condition and the controller design approach are given in term of LMI.

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REFERENCES

[1] Yue D, Han Q L, Peng C," Sate feedback controller design of networked control systems," IEEE Transaction on circuits and systems, vol. 51, pp.: 640- 644, 2004.

[2] Hejun Yao, "Exponential stability control for nonlinear uncertain networked systems with time delay," Journal of Discrete Mathematical Sciences and Cryptography, vol.21, pp.563-569, 2018.

[3] Gao H, Chen T, "A new delay system approach to network-based control,"Automatica, vol.44,pp. 39-52,2008.

[4] Hejun Yao, "Finite-time Stabilization for a Class of Networked Systems with Delay," The Open Automation and Control Systems Journal, vol.6, pp. 1779-1784, 2014.

[5] Xiong Y S, Yu L, Xu J M, "Design of sliding mode predicting controller for networked control system," Electric Drive Automation,vol.25, pp. 39-40, 2003.

[6] J. G. Li, J. Q. Yuan, J.G. Lu, "Observer-based H_∞ Control for Networked Nonlinear Systems with Random Packet Loss," ISA Transactions, vol.49, pp.39–46, 2010.

[7] H. Gao, "Stabilization of networked control systems with a new delay characterization," IEEE Transactions on Automatic Control, vol. 53, pp. 2142-2148, 2008.

[8] D. Huang, "Dynamic output feedback control for uncertain networked control systems with random network induced delays," International Journal of control, Automation, and Systems, vol. 7, pp. 841-847, 2009.

[9] Y. Wang, " $\boldsymbol{H}_\infty$ control of networked control systems with time delay and packet disordering," IET Control Theory Appl., vol. 1, pp. 1344-1354, 2007.

[10] X. Jiang, "A new H_∞ stabilization criterion for networked control systems, IEEE Transactions on Automatic control, vol. 53, pp 1025-1031, 2008.

[11] Hejun Yao, "Guaranteed cost control for discrete uncertain time delay networked systems with sliding mode approach," Journal of Intelligent and Fuzzy Systems, vol.35, pp.3959-3969, 2018.